Constant-net-time headway as a key mechanism behind pedestrian flow dynamics

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We show that keeping a constant lower limit on the net-time headway is the key mechanism behind the dynamics of pedestrian streams. There is a large variety in flow and speed as functions of density for empirical data of pedestrian streams obtained from studies in different countries. The net-time headway, however, stays approximately constant over all these different data sets. By using this fact, we demonstrate how the underlying dynamics of pedestrian crowds, naturally follows from local interactions. This means that there is no need to come up with an arbitrary fit function (with arbitrary fit parameters) as has traditionally been done. Further, by using not only the average density values but the variance as well, we show how the recently reported stop-and-go waves [Helbing *et al.*, Phys. Rev. E **75**, 046109 (2007)] emerge when local density variations take values exceeding a certain maximum global (average) density, which makes pedestrians stop.

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I. INTRODUCTION

With an increasing population and with more cost effective transportation, mass gatherings become more frequent. The total size of such gatherings are often as large as millions of people, for example, during the inauguration ceremony of president Obama [1] and the Hajj pilgrimage to Mecca [2].

To guarantee the safety of the participants during such large mass gatherings, careful planning needs to be carried out by the organizer. During the last decades, numerous empirical studies [2–13] have been performed on pedestrian crowds in different countries in order to understand the dynamics of these crowds. Even though an understanding of crowd dynamics is a prerequisite for being able to plan a mass gathering, there is still no consensus on some of the most basic relations, such as how the flow of people (people per meter per second) depends on the crowd density (people/m^2). Misconceptions of these basic relations may result in serious safety risks during mass gatherings [13].

Let us now start from the bottom up, and show how local interactions lead to certain flow-density relationships for the stream of pedestrians. Since movement and avoidance patterns of pedestrians tend to be rather complex, the traditional way to reduce complexity is to find a relation of the flow Q ($\mathrm{m}^{-1}\ \mathrm{s}^{-1}$) as a function of the average density ϱ (m^{-2}). Using this relation, called the *fundamental diagram*, has been successful to some extent, but unfortunately there are large variations in these relations, among empirical studies carried out in different countries. All these studies agree on that walking speed of pedestrians is a decreasing function of density, but they disagree on how this function looks like. We will now demonstrate how the net-time headway, as a result of finite reaction times, is the key mechanism which can explain the discrepancies between data sets from different studies.

II. REACTION TIME

It is known from traffic science that finite reaction times are needed to explain instabilities in traffic flows [14]. For pedestrian-flow dynamics, the role of finite reaction times has not been investigated in detail. By doing so, it turns out that the finite reaction time gives rise to a certain net-time headway, which is needed as a safety headway, to prevent accidental physical encounters with surrounding pedestrians.

Many-particle simulations [15,16] coupled with empirical pedestrian-trajectory data [17] reveal the probability-density function of delay times T^d from a walking experiment [18] where two pedestrian streams are intersecting at a 90° angle. The resulting distribution of delay times are shown in Fig. 1.

Interestingly, the probability-density function of the delay curve is bimodal. The first peak occurs at lower times than the typical response times, to visual or acoustic cues [19,20]. Therefore, this peak must correspond to *anticipated* movements of the surrounding pedestrians. The second peak at around 0.45 s occurs at times which are significantly larger than the previously mentioned response times but also lower than response times involving conscious reactions [14,21]. Therefore, we conclude that this second peak corresponds to an unconscious response, which is more complex than a simple reaction. In fact, it has been shown that reactions

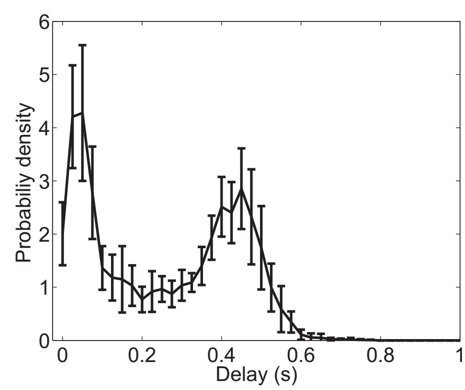

FIG. 1. Probability-density function of the delay T^d. The error bars correspond to one standard deviation. Interestingly, the probability-density function is bimodal. When the surrounding pedestrians are acting in a way that is easy to predict, extrapolation allows to anticipate their behaviors, while a delayed reaction results in cases of unexpected behaviors.

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where there are more than one possible response (choice reaction time) as well as reactions to more complex cues (recognition reaction time) take significantly longer time [22].

We interpret the bimodality as follows: when the surrounding pedestrians act in a way that is easy to predict, extrapolation allows to anticipate their behaviors, while a delayed reaction results in cases of unexpected behaviors.

III. MODEL

There has been a rich amount of microscopic models of pedestrian dynamics, for example, the social-force model [15,16] and cellular-automata models [23–25]. These models are able to reproduce various self-organization phenomena, such as lane and stripe formation [26], freezing by heating [27], Mexican waves in excitable media [28], intermittent outflows [29], stop-and-go waves, and crowd turbulence [15].

When measuring empirical pedestrian flows and densities and then fitting a suitable curve to the data, one obtains a function which is useful for engineering involved in planning of pedestrian facilities. This pragmatic fit curve, however, does not provide any insight into the mechanisms and dynamics behind the pedestrian interactions and behaviors, leading to the aggregated data.

However, when plotting the fundamental diagrams obtained in various empirical studies [Fig. 3(a)], one can see that each of the curves has a similar parabolalike shape. Nevertheless, the curves are quite different from one measurement site to another. One question remains to be answered: what, if any, is the common underlying principle of these curves?

In an attempt to bridge this knowledge gap, let us come back to the issue of reaction times, mentioned before. Since pedestrians have a typical reaction time $T^d=0.45$ s to unexpected behaviors of surrounding pedestrians, it would be natural that they compensate the risk of bumping into others, by keeping a certain safety time headway to the surrounding pedestrians [30].

To connect the aggregated density to local interactions, let us approximate the mean distance between the center of masses of a pedestrian α and the closest pedestrian β by $d=\langle d_{\alpha\beta}\rangle=1/\sqrt{\varrho}$, where ϱ is the global [31] (average) density. Note that this would hold only if the pedestrians were distributed into a square lattice, but for other density distributions, it will serve as a fair approximation (see Fig. 2).

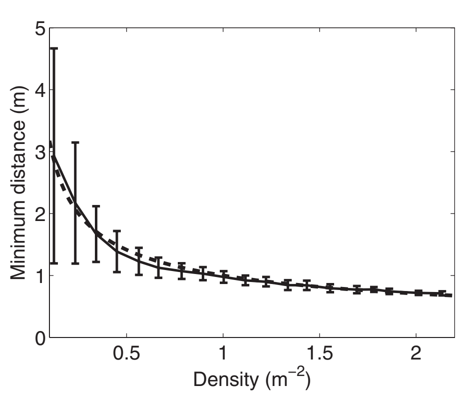

FIG. 2. The distance between an arbitrary pedestrian α and the closest surrounding pedestrian β as a function of global (average) crowd density ϱ. The solid line shows the average value ± one standard deviation as error bars, and the dashed line shows the fit curve $1/\sqrt{(\varrho)}$. The data is from Ref. [18].

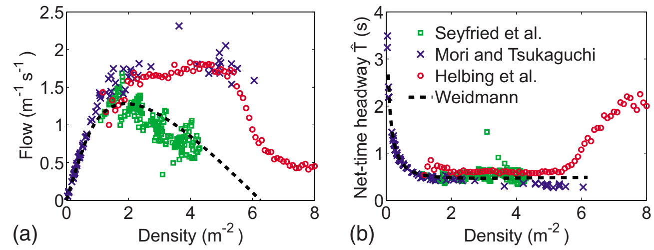

FIG. 3. (Color online) (a) flow as a function of density for data from a number of empirical studies. (b) the net-time headways $\hat{T}$ as a function of density ϱ. $\hat{T}$ is most often bounded by a constant lower value of about 0.5 s. In the data of Ref. [2], however, there is a transition for high densities where $\hat{T}$ suddenly increases. The data sets are the same as used in Ref. [2], i.e., the data from (Helbing *et al.*) correspond to *local* densities and flows.

The net distance is defined as $\hat{d}=d-2r$, where $r=1/(2\sqrt{\varrho_{max}})$ is the effective radius of a pedestrian and ϱ_{max} is the largest measured density.

Assuming that the predecessor β [32] would suddenly stop [29], it would take $\hat{T}=\hat{d}/v_\alpha=(1/\sqrt{\varrho}-1/\sqrt{\varrho_{max}})/v_\alpha$ seconds before a physical encounter with pedestrian α occurs if v_α is the speed of pedestrian α. We now show how the net-time headway $\hat{T}$ depends on the global density ϱ by applying the above scheme to empirical data determined from different studies [see Fig. 3(b)].

Note that $\hat{T}$ saturates at a constant value that is very similar to the response time to unexpected behaviors (see Fig. 1). However, in the data of Ref. [2] there is a transition at very high densities, where $\hat{T}$ suddenly *increases*. This can be interpreted in at least two ways:

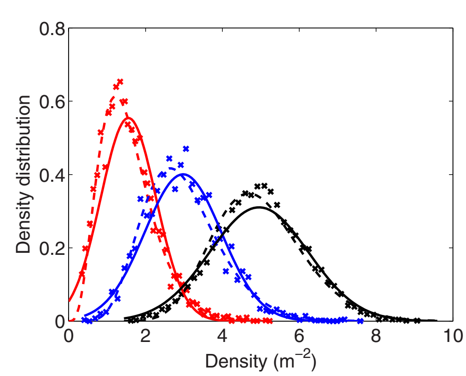

FIG. 4. (Color online) Distributions of local densities ρ for three different global (average) densities $\varrho=1.6$, 3, and 5 pedestrians/m^2. The data comes from Ref. [2]. For each global density ϱ a Beta distribution is fitted (dashed lines). However, Gaussian distributions (solid lines) also fit the data fairly well. The Gaussian distributions are produced with the parameters $\mu=\varrho$ and $\sigma=\sqrt{\varrho/3}$.

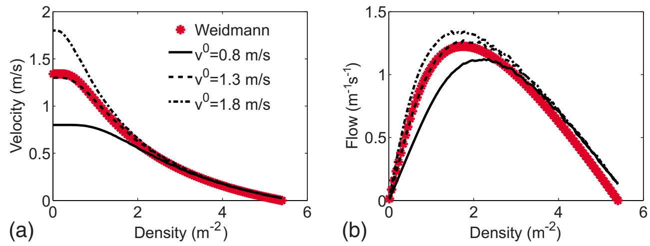

FIG. 5. (Color online) Fundamental diagrams and velocity-density relations generated by Eq. (1), for different free speeds v^0. Note how they all converge for large densities. As a comparison, the empirical fit curve by Weidmann [3] is shown.

(i) *Hypothesis 1*. When the density is very high, pedestrians start to have fear of crushing or asphyxia [33], and therefore want to increase the space around themselves (leading to higher net-time headways $\hat{T}$).

(ii) *Hypothesis 2*. If the space in front of a pedestrian is too small (or the velocity is too low) it will no longer be possible to take normal steps. Rather, pedestrians will completely stop until they have gained enough space to make a step.

In previous work [33], hypothesis 1 has been used. In this study, however, we will investigate hypothesis 2. This interpretation would naturally explain the empirically observed

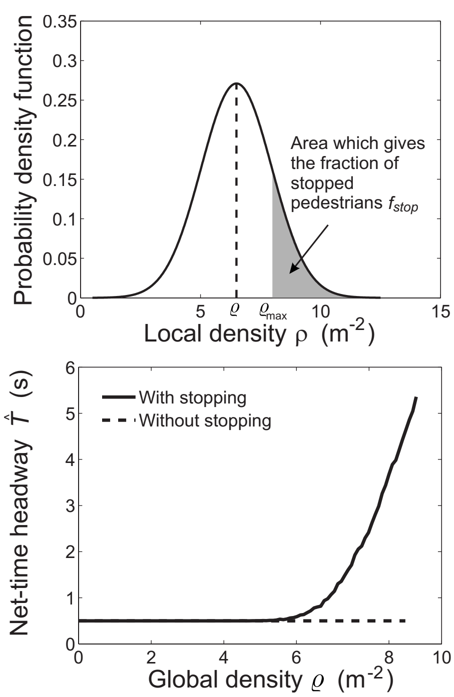

FIG. 6. (Top) the mean net-time headway $\langle\hat{T}\rangle$ is obtained via the fraction of pedestrians who are physically colliding with others and is therefore stopping and temporarily increasing their net-time headway. This fraction is obtained by integrating over the probability-density-function of the local-density distribution, starting at local densities ρ that are higher than the maximum global density ϱ. (Bottom) the mean net-time-headway $\langle\hat{T}\rangle$ (solid line) as a function of the global density ϱ. The net-time headway without stopping is displayed as a dashed line.

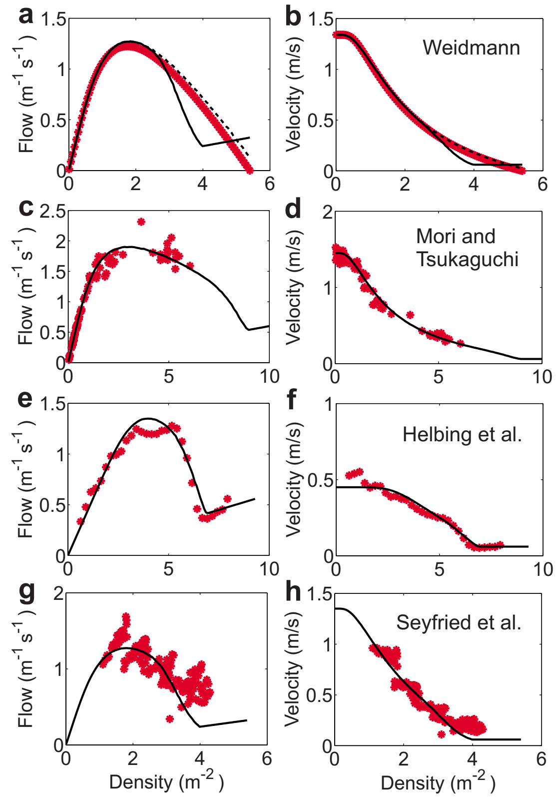

FIG. 7. (Color online) Fundamental diagrams and velocity-density relations generated by Eqs. (1)–(3), assuming a constant-net-time headway $\hat{T}=0.5$ s and a minimum velocity $v_{min}=0.06$ m/s. Red markers represent empirical data and solid lines the theoretically expected relationships. (a) Flow-density data, and (b) velocity-density data by Weidmann [3], compared to a fundamental diagram generated with the parameters $\varrho_{max}=5.4$ m^{-2} and $v_{max}=1.34$ m/s. The dashed line shows the result when it is assumed that no pedestrians are stopping, i.e., $f_{stop}=0$. (c) Flow-density data and (d) velocity-density data by Mori and Tsukaguchi [4], compared to a fundamental diagram generated with the parameters $\varrho_{max}=12$ m^{-2} and $v_{max}=1.45$ m/s. (e) Flow-density data and (f) velocity-density data from Helbing *et al.* [2], compared to a fundamental diagram generated with the parameters $\varrho_{max}=9.3$ m^{-2} and $v_{max}=0.45$ m/s. (g) Flow-density data and (h) velocity-density data from Seyfried *et al.* [11], compared to a fundamental diagram generated with the parameters $\varrho_{max}=5.4$ m^{-2} and $v_{max}=1.35$ m/s.

stop-and-go waves analyzed in Ref. [2] and would further imply: above an average density of 5 persons/m^2, the fundamental diagram will no longer describe the dynamics of the crowd well since the flow rate is then alternating between movement and standstill rather than continuous.

In an attempt to unify all fundamental diagrams in the same framework, the following scheme is proposed:

Each pedestrian α has a free speed $v_\alpha^0=v_{max}$ (which is an upper speed limit, occurring when $\varrho\rightarrow 0$). Each pedestrian also has a lower limit v_{min} of the speed. For $v<v_{min}$, pedestrians can no longer make normal steps, and would rather stop completely. For simplicity, these values are assumed to be the same for all pedestrians. It has been reported in Ref. [34] that the free (unconstrained) headways are exponentially

distributed, where the constrained headways on the other hand, are limited by a desired minimum headway. Therefore, we propose that the net-time headway $\hat{T}$ is the key control parameter for the fundamental diagram. That is, pedestrians will decrease their speeds, if necessary, to assure a constant lower limit of the net-time headway $\hat{T}$. The fundamental diagram can now be specified as

$$v(\varrho) = \frac{d-2r}{\hat{T}} = \frac{1/\sqrt{\varrho} - 1/\sqrt{\varrho_{max}}}{\hat{T}} \quad (1)$$

and bounded by $[v_{min}, v_{max}]$

It has been shown in Ref. [35] that each average density ϱ corresponds to a distribution of local densities ρ, and we therefore approximate the density distribution with a Gaussian distribution $\mathcal{N}(\varrho, \sqrt{\varrho/3})$ with mean ϱ and standard deviation $\sqrt{\varrho/3}$ (see Fig. 4).

According to hypothesis 2, defined above, we get an extra constraint, saying that pedestrians will stop walking if they are too close to other pedestrians, which happens for $\rho \geq \varrho_{max}$ (physical interaction). They will then resume walking again when they have enough space L for taking a step. Since one step (for low walking speed) needs approximately $L \approx 0.5$ m [34], we get a new net-time headway $\hat{T}' = L/v_{min} \approx 10$ s whenever $\rho \geq \varrho_{max}$.

The fraction of pedestrians that are physically colliding with others can be measured by integrating the probability-density-function of the Gaussian distribution [see Fig. 6 (top)]

$$f_{stop} = \int_{\varrho_{max}}^{\infty} \mathcal{N}(\rho) d\rho, \quad (2)$$

with mean $\mu = \varrho$ and standard deviation $\sigma = \sqrt{\varrho/3}$. Then, the mean net-time headway [see Fig. 6 (bottom)] is given by the fraction of stopped pedestrians as

$$\langle \hat{T} \rangle = (1 - f_{stop})\hat{T} + f_{stop}\frac{L}{v_{min}}. \quad (3)$$

Figure 5 shows generated fundamental diagrams from Eq. (1) with the parameters $\hat{T} = 0.5$ s, $\varrho_{max} = 5.4$ m^{-2}, and for different values of the free speed $v^0 = v_{max}$. Since v^0 only gives the upper limit of the velocity, fundamental diagrams with different v^0 converge at high enough crowd densities, given that all other parameters are fixed. The reason is that, for high density, the movement is transformed from individual walking to walking which is constrained by other pedestrians.

We now apply the method outlined above on different empirical fundamental diagrams. In all cases we use $\hat{T} = 0.5$ s and $v_{min} = 0.06$ m/s. Starting with Weidmann's [3] fundamental diagram, we have the parameters $\varrho_{max} = 5.4$ m^{-2} and $v^0 = 1.34$ m/s, which is displayed in Figs. 7(a) and 7(b) together with our curve, obtained by Eqs. (1)–(3).

Next, we apply our method to the fundamental diagrams of Refs. [2,4,11] and obtain the results presented in Figs. 7(c)–7(h). The fit functions match all four different empirical data sets well. All parameters are kept constant over the different data sets, except the maximum density and the free speed, but these two values are obtained from the data rather than tuned in order to fit the data.

IV. CONCLUSIONS

The constant-net-time headway is a natural safety mechanism to compensate for the reaction time to unexpected events. It has been demonstrated that various data sets, from different countries, all share the same net-time headway $\hat{T} = 0.5$ s. The particular advantage of our method is that it follows naturally, without the need of an arbitrary fit function. Further, all the parameters are measurable, such as the free speed and the maximum density. There is not a single free parameter that must be tuned in order to fit the different data sets. However, it should be mentioned that even though the maximum density can be estimated, it can normally not be exactly determined from the data. This is addressed in recent work [36] that may make it possible to obtain culturally dependent parameters, such as the maximum density.

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